# FROM DIVINE ORDER TO HUMAN APPROXIMATION: MATHEMATICS IN BAROQUE SCIENCE

**DRAFT**

OFER GAL

UNIVERSITY OF SYDNEY

## PART A: THE ORBIT (THE UNRULY TRAJECTORY FROM ABSURD COUNTERFACTUAL TO MANAGEABLE FACT)

### 1. KEPLER AND NEWTON

"… Here ponder too the Laws which God / Framing the universe, set not aside / But made the fixed foundations of His work" (Halley's *Ode to Newton*)

"The catechism reveals God to children, but Newton has revealed him to the sages." (Voltaire) [more citations]

This is the way in which Newton and his disciples wanted his great achievement to be remembered: as the submission of all phenomena to a small set of exact mathematical laws. These laws, Halley and Voltaire avowed, constituted a simple, perfect and harmonious structure underlying all seemingly unruly phenomena, a structure which was the divine blue print for the universe. They had to be mathematical because mathematics was the way the catechism was revealed to the sages; the science of simple, perfect structures, human reason in closest approximation of the divine[1].

[1] On the Jesuits use of these ideas in their attempts to convert the high cultures of East Asia see *Journey to the East*.

The two diagrams below suggest that this self presentation was not unfounded. They suggest that the assumption that simplicity of causes must underlie the complexity of phenomena, and that deciphering this simplicity is the role of mathematics, were not only philosophical afterthoughts for Newton and the Newtonians. Rather, they were working principles that he picked up from the tradition of mathematized natural philosophy developed through the 17th century and which Kepler was instrumental in shaping:


*Figure 2: Kepler's* Astronomia Nova

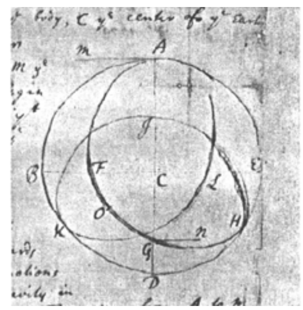

*Figure 1: Newton's letter to Hooke*

70 year separate these diagrams, and they are different in audience and target. Kepler's is public and in print—it opens the 1609 *Astronomia Nova*—Newton's is private and hand-drawn—part of a 1679 letter to Robert Hooke. Kepler is aiming to convince the general astronomical public that the geostatic system, whether in its Ptolemaic or Tychonic version, is untenable. Newton is suggesting to his correspondent Hooke that his—Hooke's—idea that planetary motions are a compound of inertial motion and solar attraction is fundamentally flawed. Kepler's diagram is based on a careful calculation from the geostatic theory he thinks obsolete; Newton, on the other hand, feigns a quantitative theory he does not really have and fabricates a construction[2].

Yet the structure of the argument these diagrams embody is essentially the same, and very much in line with Halley and Voltaire's pronouncements. Both depict a hypothetical planetary orbit, suggested

[2] Naunberg (ref) claims that Newton is basing his claim on a calculation. There is no evidnce for that claim. C.f. De Gandt..

by the theory under consideration, and both expect their audience to immediately perceive the orbit as obviously absurd, and eschew the theory that produced it. And why is the orbit obviously absurd? Because it is **chaotic**. Because, quoting Kepler, "these motions, continued farther, would become unintelligibly intricate, for the continuation is boundless, never returning to its previous path" (ref.) As Newton will put it later, Hooke's idea means that "there are as many orbits to a planet as it has revolutions." And for both writers the argument ends here—an "unintelligibly intricate" orbit is ***prima facie*** unacceptable.[3]

An absolute trust in simplicity and orderliness is entailed in these diagrams. This is such a strong assumption, that it needs not explication: the reader—Hooke or the general readership—is expected to accept the impossibility of the theory by merely looking at the complexity of the orbit. And it is so strong, that it seems to spill over from the level of causes to that of phenomena[4]: it is the orbit that is complex, complain Kepler and Newton, but it is complex enough not to allow a simple cause.

However, I intend to demonstrate, the idea of Newton's approach to mathematics we inherited from the likes of Halley and Voltaire is incorrect. These twin beliefs; in simple harmonies and in the power of mathematics to discover them, were Kepler's, but no longer Newton's. For Kepler, the diagram and the argument it supports represented a genuine commitment: these beliefs guided him and were embedded in his work throughout his career. For Newton, on the other hand, they were already, or soon to become a commonplace. He could use this type of 'argument from order' effectively, but by the time he sends his sketch to Hooke (1679-80), it represents no more than a rhetorical *topos*, which he easily forgoes once it conflicts with his problem-solving strategies. For all

[3] Ref to Ashgate

[4] To be exact, to the level of 'theoria': in traditional astronomy this is status of the plotted orbit.

practical intents and purposes, Newton's universe is imperfect and far from simple. For Kepler it is the perfection of mathematics which makes it the proper medium through which to express the beauty, majesty and absolute perfection of the Creator and His creation—the magnificent Harmony of the World. In diametric opposition, Newton's work takes the turn that would lead him to the *Principia* once, following the correspondence of Hooke, he adopts the view expressed in the following Scholium:

> The whole space of the planetary heavens either rests … or moves uniformly in a straight line, and hence the communal centre of gravity of the planets … either rests or moves along with it. In both cases … the relative motions of the planets are the same, and their common centre of gravity rests in relation to the whole of space, and so can certainly be taken for the still centre of the whole planetary system. Hence truly the Copernican system is proved a priori. For if the common centre of gravity is calculated for any position of the planets it either falls in the body of the Sun or will always be very close to it. By reason of this deviation of the Sun from the centre of gravity the centripetal force does not always tend to that immobile centre, and hence the planets neither move exactly in ellipse nor revolve twice in the same orbit. So that there are as many orbits to a planet as it has revolutions … and the orbit of any one planet depends on the combined motion of all the planets, not to mention the action of all these on each other. But to consider simultaneously all these causes of motion and to define these motions by exact laws allowing of convenient calculation exceeds … the force of the entire human intellect. Ignoring those minutiae, the simple orbit and the mean among all errors will be the ellipse …[5]

Many interesting observations can be offered about this scholium. Not the least of them is that this Scholium has been carefully omitted from his *opus magnum*, in which the rhetoric of perfection and accuracy is carefully maintained (citation from the 3rd book). This is particularly telling because the scholium is central to Newton's *De Motu Sphæricorum Corporum in Fluidis*—one of the last drafts of the *Principia*; it has clear merit as a summary of Newton's mechanical cosmology; and Newton's search for such summaries[6].

The discrepancy between practical acknowledgement of irreducible complexity and the insistent public avowal of discoverable, fundamental

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[5] ref

[6] I. B. Cohen Introduction to the *Principia* about the version of Newton's 'book 2'.

simplicity is a cultural phenomenon of utmost importance; part of the legacy of early modern science that it modern successor has never reflected on. But my interest in the following is not the in the discrepancy but the practices. I will try to demonstrate that the sentiment expressed by Newton in the scholium is a genuine expression of the metaphysical commitment embedded in his work, to which he is fully aware. In other words, the accolades Newton coaxed from his followers capture what Kepler was aiming at, but not what Newton finally achieved.

Two additional points have to be noted concerning this scholium. The first is that it is not skepticism that Newton suggests. It is true, "to consider simultaneously all these causes of motion and to define these motions by exact laws allowing of convenient calculation exceeds … the force of the entire human intellect." The import of this declaration, however, is not a lamentation of the limits of "the force of the entire human intellect." It is, rather, a short marker of what this intellect is capable of and should be expect to provide: "Ignoring those minutiae, the simple orbit and the mean among all errors." It is within our powers to decide what to "ignore." Newton is rarely suspected of skepticism; my point is not his unquestioned epistemological confidence but the fact that he bases it in this scholium on our active capacity to mathematically approximate a "mean among all errors." The other point is that, like the two diagrams above, the discussion seems to be on the level of phenomena—of the motions of the planets—and one may read Newton as implying that there is, behind these phenomena, a level where "exact laws" rule. That the scholium narrates the effects of "centripetal forces" around a "center of gravity" suggests that by "exact laws" Newton refers to the inverse square law (ISL) as "the fixed foundations of His work," but at least in this scholium, this is not what Newton is advocating. There are "causes" in the plural and the laws are to "*define* these motions" affected by them. These are human made laws that Newton refers to here, designed for "allowing of convenient calculation." At least in this scholium, the world painted by Newton is dominated by the specter of "as many orbits to a planet as it has revolutions," and his promise is not to find their "foundations" but only "the mean among all errors." The ISL, I will try to demonstrate, is such a mean.

## 2. Kepler and Perfection

The dream of "foundations" could of course be very real. For Kepler, the aspiration to arrive, through mathematics, at the simple, divine infrastructure of our world was a genuine commitment. The trust in universal harmony and an effective belief in the power of mathematics to discover it is not merely a metaphysical, epistemological or religious position for him, but a working strategy. It is a 'working metaphysics', assumed in the argument based on the *Astronomia Nova* diagram and embedded in this work throughout (example). In his *Mysterium Cosmographicum,* published 13 years earlier, Kepler provides his most explicit expression of both: the universe, he tells his readers, is "complete, thoroughly ordered and most splendid" (23-4/95-97). It is simple, and its structure necessary. Kepler's mathematical inquiry is strictly structured by these assumptions. His question in the *Mysterium* is why there are exactly six planets, and his answer is that there are exactly five perfect solids. Thus, if the distances between the planets correspond to these solids, namely—if the proportions between their distances can be shown to correspond to the proportions between the solids (for there are no material solids in the heavens)—then the number of planets has been explained—the mathematical directly account for the physical.

But what kind of an explanation is this? Why should abstract mathematical proportions account for a material fact? Why should their aesthetic value be evidence for their truth? Guided by the metaphysics of order, Kepler suggests two complementary answers to this question. Either:

> God, like one of our own architects, approached the task on constructing the universe with order and pattern, as if it were not art which imitated Nature, but God himself had looked to the mode of building of Man who was to be

Or:

> it is by some divine power, the understanding of the geometrical proportions governing their courses, that the stars are transported through the ethereal fields and air free of the restraints of the spheres

Either mathematics is God's own blueprint for the universe, or the planets themselves are using the "geometrical proportions governing their courses" to navigate the empty vastness of the heavens. Kepler never gives up on the first possibility (that he would later use as a proof for creation and the existence of God). However, it is and awkward assumption, that the rationality of the structure can only affect the material realm if that realm (or elements of it, like the celestial bodies) is also endowed with rationality. Kepler largely retreats from it in the *Astronomia Nova*. But, against common wisdom, Kepler never eschews the mathematical enthusiasm of the *Mysterium* for the physicalism of the *Astronomia Nova*. This is how he himself analyses his development in a note he adds to this paragraph in the second edition of the *Mysterium*, published unchanged (apart from the annotation) in 1621: "So indeed I supposed," he says, concerning the rationality of the planets,

> but later in my *Commentaries on Mars* [the *Astronomia Nova*] I showed that not even this understanding is needed in the mover. For although definite proportions have been prescribed for all the motions … by God the Creator, yet those proportions between the motions have been preserved … not by some understanding created jointly with the Mover, but by … the completely uniform perennial rotation of the sun [and] the weights and magnetic directing of the forces of the moving bodies themselves, which are immutable and perennial properties. (60/169)

Note what it is that Kepler thinks he has defended in the Astronomia Nova. The notion of perfect proportions remains untouched; changed is only the mechanism by which they are followed. The mechanism itself is simple: the rotation of the sun and the magnetism of the planets are "completely uniform [and] perennial." The mathematics—the analysis of the proportions between the solids and the consequents distances and periods—is left to safeguard the "complete, thoroughly ordered and most splendid universe."

## 4. Newton and the Moving Apsides

It does not mean, of course, that Kepler is unaware of the difficulties in applying his grand mathematical scheme to the minute details of observation. Quite the contrary. The major part of the *Mysterium* is dedicated to this task; in particular, to finding a place for the eccentricities of the planetary orbits between the nesting polyhedra—this was the pretext for his correspondence with Brahe, leading to their illustrious collaboration. The point, however, is exactly this: Kepler excuse the eccentricities by fitting them into the mathematical model constructed according to independent principles. For Kepler, the world has a universal, harmonic, perfect structure, which can be discovered by a-priori, mathematical considerations, and into which one then has to fit the embarrassing particularities of the empirical[7].

Compare now Kepler's genuine pursuit of "Laws which God / Framing the universe, set not aside" to Newton's transformation of the argument against Hooke, which in its original form was so similar in structure to Kepler's. Its final version is to be found in Propositions 43-45 of the first book of the *Principia*:

> If a body, under the action of a centripetal force that is inversely as the square of the height, revolves in an ellipse having a focus in the center of forces, and any other extraneous force is added to or taken away from this centripetal force, the motion of the apsides that will arise from that extraneous force can be found out … and conversely. (Proposition 45, Corollary 2, *Principia*, 544)

Newton turns on its head his previous suggestion that the consequent motion of the apsides invalidates Hooke's proposal to "compound the celestiall motions of the planets." Now, it is the ability to calculate this

[7] C.f. Stephenson's *Kepler's Mathematical Cosmology*

motion that assures him of the power of his mathematics and the validity of his physical assumptions.

But this new ability hinges on changing the very notion of an orbit and the way mathematics is used to construct it, and the change implies that the assumptions of perfection and simplicity underlying the original argument had been abandoned. This is borne in the particulars of Newton's mathematics.

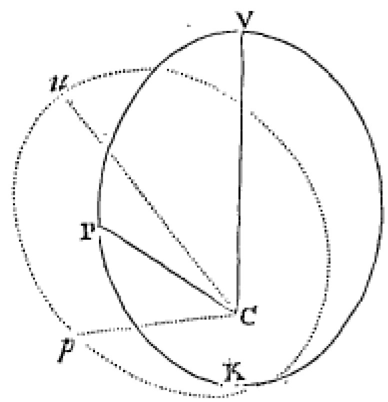

*Figure 3: Proposition 43*

The crucial change transpires when the moving apsides turn from a mark of a non-orbit to property of a revolving orbit. Newton affects this in Proposition 43 by teaching his readers "to find the force that makes a body capable of moving in any trajectory that is revolving about the center of forces in the same way as another body in the same trajectory at rest" (*Principia*, 534). Given orbit VPK with center of force at C, Newton shows how to construct for each point P on this closed curve (the diagram suggests it is a Keplerian ellipse, but nothing in the proof refers to or depends on this) an identical curve through *up* at a constant angle PC*p*, equal to VC*u*. Now, since VPK is an orbit around C, namely P is moving about C towards which it is drawn, it follows that area VCP is proportional to time. This is the very first theorem Newton prove in the *Principia* (Proposition 1 Theorem 1; *Principia*, 444-446). It is Kepler's law of areas, his so-called 2nd law, generalized from an empirical approximation into a mathematical truth about all bodies revolving around a center of force[8]. Additionally, by the conditions of the propositions, angle VC*p* is proportional to angle VCP. Hence, the area described by C*p* will be proportional to area VCP, and therefore proportional to time.

[8] On the import of Newton's proof of Kepler's area law see De Gandt …

The proof that C*p* is sweeping areas proportional to time is what turns “the figure *u*C*p*” into a segment of an orbit around C. This follows from Newton’s second theorem (Proposition 2 Theorem 2, *Principia*, 446-448), according to which “every body that .. by a radius drawn to a point … describes areas around that point proportional to time, is urged by a centripetal force tending toward that same point.” In other words, not only does every body orbiting around a center of force abide by the area law, the complementary is also true: every body abiding by the area law around a given point is describing an orbit around that point as a center of force, so *u*C*p* is an orbit:

> the body, being always at *p*, will move at a perimeter of the revolving figure *u*C*p*, and will describe its arc *up* in the same time in which another body P can describe the arc VP, similar and equal to *up*, in the figure VPK. (*Principia*, 534).

Proposition 43 is pivotal not only because it allows Newton to claim control over the concept of a revolving orbit, but also because it formulates the difference between the stable and the revolving in terms of the difference. In the next proposition he calculates exactly what this is:

> The difference between the forces under the action of which two bodies are able to move equally—one in an orbit that is at rest and the other in an identical orbit that is revolving—is inversely as the cube of their common height. (Proposition 44, *Principia*, 535)

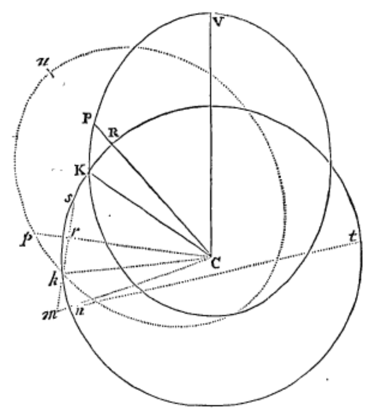

*Figure 4: Proposition 44*

This ability to calculate the difference in force will be Newton’s knockdown argument for the gravitation between the sun and the planets following the ISL, and we will return to it below.

The transformation of the unruly, hence impossible trajectory of the letter to Hooke into an orbit is thus an impressive show of skill and an important achievement—it is the culmination of Newton’s treatment of “motion of bodies in orbits whose planes pass through center of forces” (*Principia*, 545), after which he moves on to deal with oscillation.

But what exactly has been achieved? C*p* obeys the areas law simply by virtue of C being a center of force, and "figure *u*C*p*" is an orbit by virtue of obeying the areas law. But the body at *p* does not travel along the dotted line. Newton makes it very clear in his illustration to Proposition 44 (*Principia*, 537): the body at *p* does not continue to *k* but to *m*. *up* is not a real trajectory—it is only a copy of VPK around one known point *p* of the real trajectory. The real trajectory, like that in the drawing sent to Hooke, has no regular line of apsides. Moreover—VPK itself is not a real trajectory; there is no body that moves on that curve. Both trajectories—at rest and revolving—are mathematical fictions. The orbit itself, the real trajectory in which the body is travelling does not possess any mathematical status; it is not a recognizable curve and Newton does not presume to draw it. One may say that Newton returns the real trajectory to its pre-Kepler status of mathematical and theoretical irrelevance; "there are as many orbits to a planet as it has revolutions." Instead we have points like V and *p*—presumably locations to be determined from observations—and fictive orbits constructed for calculation purposes. Of course, the end of the calculation is a brilliant realization of Kepler's hope for *physica coelestis*: the mathematics provides causal account, relating real bodies by forces that impact their motions. But the justification for this hope, the idea of simple mathematical infrastructure, has disappeared. Mathematics is not embedded into the behavior of the revolving body; it is only a sophisticated means to decipher it.

Propositions 43-45 put regularity into trajectories which follow no regular curve and have no stable line of apsides—they turn them into orbit. But it is *artificial* regularity; it is the work of art, on the construction of fictive orbit. This regularity is the assurance that some features of this orbit can be determined by reason, but the determination comes by the application of art rather than by the discovery of rational or simple foundations.

## Part B: The Force Law (The ISL from Geometrical Necessity to convenient approximation)

### 5. Kepler's ISL

Kepler's planets required a rational orbit they could follow. Even when they followed those orbits mechanically, rather than by their own navigation, the orderly curve they were to draw in the heavens served as the equivalent of a final cause. What keeps Newton's body in its trajectory is not draw of mathematical property but a universal physical property: mutual attraction between all parts and particles of matter. True, the most fundamental law governing the relations between this attraction and cosmology—the area law—is a geometrical law. But this law promises neither underlying simplicity nor resultant order. The law is so general that it is independent of both the causing force and the caused trajectory, thus tells nothing of either. This trajectory does not have to be any recognizable curve, nor does it need to be a closed curve—it holds for bodies arriving and then retreating to infinity. (Newton about the parabola)

There is, of course, a mathematical side to that universal property of matter: the mutual attraction declines by the square of distance. It is tempting to take the ISL as the mathematical foundations Halley and Voltaire were hailing Newton for discovering. In the following, section I will demonstrate that this is not the case. The ISL never fulfills for Newton the foundational role that mathematical structures fulfill for Kepler in both their metaphysics and their physical-mathematical practices, nor does it carry for Newton any of the mathematical certainty that Kepler requires of his foundations.

The ISL is a particular case in point because it allows an unmediated comparison of the use of a mathematical law of nature in Kepler and Newton and highlights the change in the import of mathematics in the order—or ordering—of the universe. This is because it was Kepler who introduced the ISL as the ratio between light and distance in his *Optical Part of Astronomy* of 1604 (Henceforth: *Optics*)[9]:

> just as [the ratio of] spherical surfaces, for which the source of light is the center, [is] from the wider to the narrower, so is the density or fortitude of the rays of light in the narrower [space], towards the more spacious spherical surfaces, that is, inversely. For … there is as much light in the narrower spherical surface, as in the wider, thus it is as much more compressed and dense here than there. (*Ad Vittelionem*, 10)

Though embedded in traditional mathematical optics, Kepler's ISL is a mathematical-physical law of a new kind: a law which captures causal properties n strictly mathematical terms. Five years after completing the *Optical Part of Astronomy*, Kepler attempted to demonstrate the prowess of such kind of laws in the full-blown *physica cœlestis* of the *Astronomia Nova*. The import of light, the courier of the sun's powers to the planets, suggests it as a perfect analogy by which to conceptualize his *virtus motrix*—the solar force stirring the inertial planets. In the *Optics*, Kepler derives the ISL directly from the spherical expansion of light: "there is as much light in the narrower spherical surface, as in the wider, thus it is as much more compressed and dense here than there." Kepler's light is a quasi-physical entity which has "density," but its physical properties can be inferred directly from its mathematical ones because it is also a uniquely mathematical entity:

[9] For Kepler's the originality in introducing the Inverse Square Law, how it related to traditional optics and how it was converted to mechanics by Robert Hooke, see Gal and Chen-Morris, "The Archaeology of the Inverse Square Law (Parts I and II)."

> the spherical is the archetype of light (and likewise of the world); the point of the center is in a way the origin of the spherical solid, the surface the image of the inmost point, and the road to discovering. The surface is understood as coming to be through an infinite outward movement of the point out of its own self, until it arrives at a certain equality of all outward movements. The point communicates itself into this extension, in such a way that the point and the surface, in a commuted proportion of density and extension, are equal. (*Ad Vittelionem*; 19)

The physical properties of light follows directly from its mathematical attributes because light is a substantiation of geometry. It is not simply that it happens to expand spherically—light is the embodiment of sphericity. The mathematical structure of nature is the materialization of divine mathematical archetypes: "God the Creator had the Mathematicals with him as archetypes from eternity in their simplest divine state of abstraction," and through its mathematical essence, light serves as conduit of this mathematical structure into the world of matter.

For Kepler this is a solution to a problem that troubled him already in the *Mysterium* of 1596: "I shall have the physicists against me" he worries there, "because I have deduced the natural properties of the planets from immaterial things and mathematical figures" (ref. – *Mysterium*). The planets, we saw, require a mathematical path to follow, whether it is rationally or mechanically, and this mathematical structure has to be embedded in nature in a way that will make scientific and metaphysical sense. In 1619 Kepler was so proud of this solution that in a note he added to the new edition of the *Mysterium* he added that

> in the end Aristotle [would have to be] persuaded that splendid and plainly necessary causes for this matter could be derived from the harmonies as if from an archetype, [and] would accept with the fullest agreement the archetypes and, since they are ineffectual in themselves, God as the architect of the universe. (ref. – *Mysterium*).

The point is not so much Kepler's solution but the fact that he requires one; that he looks for a justification for the efficacy of his mathematics and looks for it in the metaphysics of divine simplicity and harmony—the metaphysics we learned to attribute to Newton. As discussed above, this metaphysics is not limited to reflection but determines Kepler's actual use of mathematical laws. In the case of the ISL, it provided the motivation and justification for the application of the law from optics to celestial dynamics as well as setting its limits. The spherical dissemination of agency from the sun to the planets and its decline with distance suggest that light may be the *virtus* by which the sun makes the planets, made of inert matter, move about it. But these very geometrical considerations finally convince Kepler that light can, at best, serve as analogy—it cannot be one an the same with the solar motive force. The planets' velocity is inversely proportional to their distance from the sun, not to the square thereof, he reasons. Additionally, light is dispensed spherically, and the motive force, apparently—only in the plane of the ecliptic. The geometrical make-up of the two types of solar emanation—the mathematics embedded in their nature—is different, so the motive force cannot follow the ISL, hence cannot be light.

Kepler does not attempt to apply the ISL to the calculation of the motion of the planets. He is attempting to identify and characterize the force that moves the planets, and the mathematical properties of this force are where he expects the necessity and harmony of the divine archetypes to assert themselves. The commitment to the God's geometrical "fixed foundations" dictated a fit between law and phenomena that the ISL could not provide.

## 5. The ISL after Kepler

Kepler's torturous way of legitimizing his mathematical 'physics of the heavens' failed to impress even his popularizer and most ardent admirer, Ismaël Boulliau. Perhaps his Catholicism released Boulliau from Kepler's protestant worries about God and mathematical perfections, or perhaps it was the benefit of another generation of mathematized dynamical thought, but Boulliau simply could not see the point in Kepler's vacillations. "On the rocks of these hesitations," he exclaims in the *Astronomia Philolaica* of 1645, Kepler "crushes his very astronomy into shipwreck,"[10] and suggests both geometrical and physical arguments to save the forsaken analogy between light and *virtus motrix*. Boulliau finds it almost hard to distinguish between them, using 'species'—that in the *Astronomia Nova* Kepler carefully reserves only to the solar force[11]—to denote light, and assaults every one of Kepler's diffident distinctions between the two.

Boulliau's patronizing tone suggests that he did not realize how daunting the task of making mathematics explanatory was for Kepler. He (Boulliau) never took upon himself, and his geometrical and physical speculations remained completely distinct from each other, even if adjacent. Another twenty years later, when Robert Hooke was attempting to follow Kepler's footsteps, he found himself facing very similar difficulties and, like Kepler, was restrained from making full physical use of the ISL by the geometrical considerations from which this ratio was born. Not that Hooke had much patience for neo-Platonic worries. In his 1665 *Micrographia* he seamlessly imports the ISL from light to gravity in the following parenthesized remark:

> [I say Cylinder, not a piece of a cone, because … that triplicate proportion of the shels of a Sphere, to their respective diameters, I suppose to be removed by the decrease of the power of Gravity][12]

Hooke is concerned here with the Tychonic problem of the implications of atmospheric refraction on astronomical observations, and he conducts Torricelli-style experiments in order to calculate the size and density of the atmosphere. This off-hand argument allows him to

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[10] Ref to Bouliau

[11] In his *Optics* (15) Kepler that with regards illumination and vision, "species of things" simply means light. C. f. Gal and Chen, "The Baroque Observer."

[12] Ref to Micrographia and to Gal and Chen, *HOS* .

approximate the height of the column of air above his mercury tubes: the decline of "the power of gravity" by the square of the distance[13]

means that instead of a truncated cone (in which the volume is proportional to the cube of height), he can calculate the column as a cylinder (namely—as if its volume is proportional to the height of the atmosphere).

This almost frivolous use of mathematical approximation is already quite removed from Kepler's grave hesitations about the way the perfection of his geometry reflects the perfection of creation, the way perfect geometry is distributed into the imperfect physical realm (through light), and what all this allows by way of physical-mathematical hypotheses. But Hooke's application of the ISL actually has more in common with Kepler's attitude than might be assumed. As I have shown in a different place,[14] the only justification he has for the move is exactly the geometrical analogy: like light, gravity, and with it the atmosphere, expands spherically. The image of spherical 'explosion' of agency or active principle from center towards periphery, which produced the ISL for Kepler, is exactly what is on Hooke's mind when he inquires about the behavior of light in the atmosphere and how the atmosphere itself is constituted by gravity. Like Kepler, he treats the agency as operating on the enveloping "shells" and can thus easily apply the law for the decline of light to the decline of gravity. But, again like Kepler, these very considerations prevent him from making real *physica cœlestis* use of the ISL. In 1673 he promises a

> System of the World ... answering in all things to the common Rules of Mechanics [which] depends on three Suppositions. First, That all Cœlestial Bodies Whatsoever, have an attraction or gravitating power towards their own Centers, whereby they attract not only their own parts ... but ... also ... all the other Cœlestial Bodies that are within the sphere of their activity; and

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[13] Hooke leaves it to the reader to do the calculations, but he clearly means that the decline is by the square of the distance. Otherwise his argument makes no sense.

[14] Book and Ashgate volume

> consequently that not only the Sun and the Moon have an influence upon the body and motion of the Earth, and the Earth upon them, but that [all the planets], by their attractive powers, have a considerable influence upon its motion as in the same manner the corresponding attractive power of the Earth hath a considerable influence upon every one of their motions also. The Second Supposition is this, That all bodies whatsoever that are put into a direct and simple motion, will so continue to move forward in a streight line, till they are by some other effectual powers deflected and bent into a Motion, describing a Circle, Ellipsis, or some other more compound Curve Line. The third supposition is, That these attractive powers are so much the more powerful in operating, by how much the nearer the body wrought upon is to their own Centers[15]

Hooke's mechanical "System of the World" was to be based on the suppositions of universal attraction, Cartesian inertia and a mathematical force law: "these attractive powers are so much the more powerful in operating, by how much the nearer the body wrought upon is to their own Centers." All is ready to apply the ISL for the decline of "attractive powers" with distance, the law he so easily imported from light to gravitation eight years earlier in the *Micrographia*. But Hooke declines the opportunity: "what these several degrees [of decline]" he adds, "I have not yet experimentally verified." The image of spherical 'explosion' that related gravity to light seems to him inapplicable to the notion "attractive power," as it seemed to Kepler concerning his motive force, so, like Kepler, he refrains from applying the law for decline from solar illumination to solar attraction. In spite not being truly committed to Kepler's notions of mathematical order, the similar geometrical reasoning which leads him to adopt the ISL prevents him from turning it into a flexible algebraic operator in the calculation of orbits.

[15] ref

## 7. Newton's ISL

And this is exactly what Newton does. Establishing the ISL[16] in two different ways, neither aspiring nor presuming certainty or foundational prowess, he can, "ignoring minutiae," find "the simple orbit and the mean among all errors." In the Third Book of the *Principia* and in various *scholia* and prefaces, Newton often presents the ISL as a paradigm of mathematical certainty injected into empirical investigation, and his disciples follow suit. In less public reflections like the Copernican Scholium, and more importantly in Newton's actual practice, the ISL is revealed, rather, as a contingent empirical fact, which mathematics allows to approximate and then to flexibly employ.[17]

This is far from claiming that Newton takes these procedures lightly or skeptically. Quite the opposite: they take arduous effort and reflect a firm conviction in the power of mathematics to produce reliable knowledge. The legitimacy of this knowledge, however, is founded and attained very differently than Kepler or Halley presumed it was. Newton infers the ISL, first, by plugging Kepler's 'harmonic law' into a geometrical proportion relating orbits and periods to centripetal forces. This equation, however, he only proves for circular, uniform motion. He then infers the ISL, independently, from Kepler's 'first law'; for elliptical orbits whose center of force is in one of the foci. This demonstration, however, carries the difficult stipulation that a minor deviation of the sun from the focus – well below the empirical resolution – will make it completely wrong.

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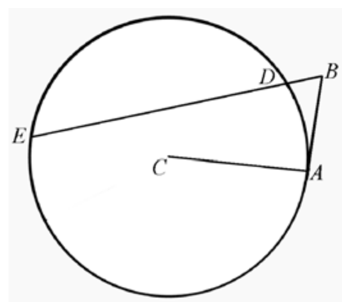
[16] He did not need to 'discover' it, C.f Gal and Chen "Archaeology".

[17] The question of the difference between practice of approximation and rhetoric of perfection here is fundamental to the understanding of late 17th century science as part of its cultural context. It deserves a full treatment in a different place.

The former way has roots in Newton's work in the 1660s[18], and it is perfected in the same version of *De Motu* from which the Copernican Scholium is taken. Using a few fast-and-loose moves which Kepler would have hardly recognized as the "Mathematicals," which "God the Creator had with him as archetypes from eternity" (*Mysterium Cosmographicum*, 1619 edition note to Ch. 11), Newton establishes a geometrical expression for the centripetal force holding a body revolving uniformly in a circular orbit: $f \propto AD^2/R$, where *AD* is an infinitesimal arc.[19] He then adds five corollaries, all simple derivations from this expression. He assumes uniform motion, so *AD* is proportional to the body's velocity. Thus, combining $AD \propto V$ with $f \propto AD^2/R$, it follows that:

Corr. 1. $f \propto V^2/R$.

Since the velocity of rotation is inversely proportional to the period of revolution, *i.e.*, $V \propto 1/T$, this is equivalent to:




Corr. 2. $f \propto R/T^2$.

Combining these two proportions, Newton can construct a force law—a ratio between force and distance—for *any* given ratio between the radius of the orbit and the period of revolution, and he demonstrates this capacity by providing three different ones:

Corr. 3. if $T^2 \propto R$, then f is distance-independent,

Corr. 4. if $T^2 \propto R^2$, then $f \propto 1/R$, and

Corr. 5. if $T^2 \propto R^3$, then $f \propto 1/R^2$.

"the case of the fifth corollary holds for the celestial bodies … astronomers are now agreed" he adds, almost as an afterthought.

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[18] Ref. to book

[19] Ref. to book.

Newton has no use for the geometrical imagery that provided Hooke and Kepler with justification for their mathematical-causal claims. He has no difficulty accepting that the physics follows the mathematics; that the actual law governing the force is simply what one finds by installing the empirical data into an abstract mathematical formula. This outcome is contingent, as Newton stresses by running through possible force laws following imaginary data. This stress on contingency is important to him: the five corollaries of the *De Motu* are expanded to nine in Proposition 4 of the *Principia*. The case of where "the periodic times are as the $^3/_2$ powers of the radii" is just the sixth of them, and the language distinguishing this particular option in the scholium to follow is hardly more excited than in *De Motu*:

> The case of corol. 6 holds for the heavenly bodies (as our compatriots Wren, Hooke, and Halley have also found independently). Accordingly, I have decided that in what follows I shall deal more fully with questions relating to the centripetal forces that decrease as the squares of the distances from centers (*Principia*, 452)

Which force law to "deal more carefully with," Newton declares, is a matter of choice; he had "decided" on the ISL. This is not a mere turn of phrase: given that the proof is limited to "the centripetal forces of bodies that describe … circles with uniform motion" (*Principia*, Proposition 4, 450), its application to the elliptical orbits and changing velocities of the primary planets *is* a difficult decision.

The point is *not* that Newton allows himself a convenient tolerance in 'massaging' the empirical data into whatever mathematic apparatus is at his disposal. Quite the contrary. Achieving "the mean among all errors" is the very task he undertakes in the Copernican Scholium, and one which he carefully defines in the previous proposition and its corollaries:

> Proposition 3, Corollary 2: And if the areas are *very nearly* (*quam proxime*) proportional to the times, the remaining force will tend towards body T *very nearly*.
>
> Proposition 3, Corollary 3: And conversely, if the remaining force tends *very nearly* toward body T, the areas will be *very nearly* proportional to the times. (*Principia*, 448-9)

Creating mathematical order in complex orbits by way of approximation is not a manner of tolerating inaccuracies or an assertion of epistemological pessimism, but a demand: the mathematics and the observations should fit *quam proxime*. Namely: if an exact force law gives an ideal orbit, an approximate one should give an orbit within the resolution of the empirical data.

Newton does not feel obliged to legitimize his physical use of mathematics the way Kepler does, but the commitments he accepts as part of this epistemology of controlled complexity also exact an unavoidable price. The *quam proxime* requirement all but prohibits demonstrating the ISL from the empirical data and Kepler's 'laws' directly, because very different laws can produce heavenly motions which are "very nearly" identical.

The problem presents itself most acutely in Propositions 10 and 11, as Newton completes his instruction of how "to find centripetal forces" (*Principia*, section 2, 444) and moves to "the motion of bodies in eccentric conic sections" (*Principia*, section 3, 462).[20] Using the same proto-infinitesimal techniques of the *De Motu* Newton proves in Proposition 10 that for a body traveling in an elliptical orbit, "the law of the centripetal force tending towards the center of the ellipse" is as the (changing) distance of the body from the center of force (*Principia*, 459). In the next proposition, no. 11, he proves that if "the centripetal force [is] tending towards a *focus* of the ellipse," it will be inversely as the *square* of the distance (*Principia*, 462-3). In other words, if the sun is in the center of the planets' elliptical orbits, gravity *increases* with distance; if the sun is at the focus of these orbits, gravity *declines as the square of this distance*. Mars' is the most eccentric of planetary orbits, and still, as calculated by Kepler, it deviates so little from the circular, the sun is *both* "very nearly"

[20] In the following I am much indebted to George Smith's excellent analysis of these theorems in "From the phenomenon of the Ellipse to an Inverse-Square Force: Why Not?"

at the center *and* very nearly at the focus. Obviously, gravity cannot be proportional to both the distance and its square.

Newton's demand that his approximation would fit '*quam proxime*' does not express a failure to apply simple mathematics to complex nature. Rather, it is a particular constraint that Newton puts on both sides—mathematics and empirical data: the mathematical law should not exactly capture an idealization of the natural motion, but to approximate a trajectory that approximates a particular curve. For the ISL to be a demonstrated law of nature, it is not enough to deduce it from Kepler's (idealized) first or third laws—the force law needs to converge towards ISL as the orbits converge to Kepler's first law. The relation between the ISL and the ellipse fails this criterion. But there is no fact of the matter as to whether the orbit is an eccentric circle or ellipse. After all, "the planets neither move exactly in ellipse nor revolve twice in the same orbit," so Newton is free to prove the ISL from the former. "The simple orbit and the mean among all errors [is] the ellipse," but simplicity is only one of the considerations which Newton applies in choosing the mathematical order to apply to nature. Given the empirical data and the *quam proxime* requirement, the ellipse does not allow to distinguish between the various possible force laws, all possible, all contingent, that could create these rather than any other orbits. There is no over-arching concept of underlying simplicity to compel Newton to accept one approximation over another.

So when writing "I have decided [to] deal more fully with … centripetal forces that decrease as the squares of the distances from centers" Newton refers to a very serious decision. It is a similar decision whether to trust Kepler's first law, lean on the proof of the ISL from the ellipse, and breach the *quam proxime* requirement, or lean on the proof of the ISL from Kepler's third law, but assume the planetary motions are in circular orbits and uniform velocities—a patently false assumption.

Newton chooses the latter. Although he is very expansive in demonstrating the capacity of his mathematics to handle orbits along various conic sections and complex curves, his work with the real planetary orbits always assumes motion "in the circumference of a circle." To do so within the within the *quam proxime* requirement, Newton develops a very complex theorem (proposition 7) which allows him "to find the law of centripetal force tending toward any given point" inside this circular orbit. Expanding it on the basis of the preceding propositions, George Smith transformed Newton's geometrical proportion into modern algebraic notation in which force is inversely as:

$$\left(\frac{SP}{a}\right)^5 + 3\left(1-\varepsilon^2\right)\left(\frac{SP}{a}\right)^3 + 3\left(1-\varepsilon^2\right)^2\left(\frac{SP}{a}\right) + \left(1-\varepsilon^2\right)^3\left(\frac{SP}{a}\right)^{-1}$$

Where *S* is the hypothetical position of the center of force (the sun in the solar system), P—the position of the moving body (the planet), a—the diameter of the orbit and $\varepsilon$—its eccentricity (the distance of center of force—the sun—from the geometrical center to the obit). As Smith acutely points out, "*SP* to the power of 2 is nowhere to be found in this expression" (40). Newton has no commitment to the ISL as representing anything beyond a convenient idealization. The expression as a whole, however, converges towards $SP^2$ the closer the eccentric circle can be seen as an approximation of an ellipse with the center of force at a focus—it provides that gravitation will be 'very nearly' proportional to $1/r^2$ if the planetary orbits are very nearly ellipses and the sun very nearly at their focus. The ISL was a feature of divine infrastructure for young Kepler—a reified 'mathematical'; it became a partially-flexible geometrical structure for Hooke; it has become sophisticated means of approximation is Newton's *Principia.*

## 8. Conclusion

In his seminal paper "Newton and the Fudge Factor" Sam Westfall argued that "not the least part of the *Principia*'s persuasiveness was its deliberated pretense to a degree of precision quite beyond its legitimate claim" (*op. cit.*, 751-2). To create this lure of precision, Westfall showed in great detail, "Newton brazenly manipulated the figures" (*op. cit.*, 755) in determining the velocity of sound and the precession of the equinoxes, and in the all important demonstration that "the attraction holding the moon in its orbit is quantitatively identical to the cause of heaviness at the surface of the earth" (*op. cit.*, 752).

This latter issue is of particular interest to us. "The law of universal gravitation," Westfall claims, "rested squarely on the correlation of the measured acceleration of gravity at the surface of the earth with the centripetal acceleration of the moon" (*op. cit.*, 752). The former was achieved by Huygens, following Mersene, by finding the length of the pendulum beating seconds. The latter Newton calculated by estimating the distance the moon would fall towards the earth in one minute. To produce the level of precision he desired, Westfall demonstrates, in this and the other examples, Newton was "doctoring the correlation" (*op. cit.*, 754).

Westfall is half bemused, half awed by Newton's audacity in "mending the numbers" (*op. cit.*, 757). The discussion above reveals, however, that the moves he calls "more public relations than science" (*op. cit.*, 755) are fundamental to the way Newton perceives the role of mathematics in the application of order to nature. Westfall accepts the textbook view that "Newton had shown that a system of planets orbiting the sun in accordance with Kepler's three laws entails a centripetal attraction towards the sun that varies inversely with the square of the distance from the sun" (*op. cit.*, 752). But Newton's System of the World is based on a 'doctored' proof. This is so both in its popular form, designed as the second book of the *Principia* but discarded by Newton for being too lenient accommodating to the unschooled, and in its final, formal version as Book Three. The claim that the force attracting the planets to the sun follows the ISL is proved by applying to Kepler's 3rd law a proportion that was only proved for uniform, circular motion. Not surprisingly, Newton's primary example for his System is provided by the moons of Jupiter, which are the most orderly of the solar system.

When Newton writes that "gravity towards the sun … decreases *exactly* as the squares of the distance as far out as the orbit of Saturn" (*Principia*, Book Three, General Scholium, 943; italics added) he is addressing the public. Nowhere in the *Principia* is such a claim supported or applied. And when he writes that this "is manifest from the fact the aphelia of the planets are at rest" (*ibid.*) he also knows he this is overstated at best. In the original *System of the World* he sufficed it with "the very slow motion of [the planets'] apses" (Newton, *System of the World*, 24), and the Copernican Scholium argues that such motion is, in principle, necessary. But this flexible and approximate use of mathematics is neither reckless nor a show of epistemological despair. It reflects exactly the way Newton perceived his science: the human enforcement of mathematical order on messy nature.

"Nature and Nature's laws lay hid in night," wrote Pope, "God said, "Let Newton be!" and all was light." What Pope had in mind was Kepler's Renaissance dream of divine order. Newton's achievement was largely indebted to relinquishing this dream in the name of the enforced order of the Baroque.

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