Solution to Selected Exercises 11

1. i

$$e^{3\ln x} = e^{\ln x^3} = x^3.$$

We used the log rule $\ln a^b = b \ln a$ and the fact that e^x and $\ln x$ are inverses.

iv

$$\ln x^3 - \ln x = \ln\left(\frac{x^3}{x}\right) = \ln x^2.$$

2. ii

$$\begin{aligned}
\ln(3+x) &= 1\\
e^{\ln(3+x)} &= e^1\\
3+x &= e\\
x &= e-3.
\end{aligned}$$

iv

$$\begin{aligned}
\ln(5x-6) &= 2\\
e^{\ln(5x-6)} &= e^2\\
5x-6 &= e^2\\
5x &= e^2+6\\
x &= \frac{e^2+6}{5}.
\end{aligned}$$

3. iv

$$\begin{aligned}
e^{x^2} &= 10\\
\ln(e^{x^2}) &= \ln 10\\
x^2 &= \ln 10\\
x &= \pm\sqrt{\ln 10}.
\end{aligned}$$

4. iii

$$\frac{1}{2}\ln(x+1) - \frac{1}{2}\ln(x) = \ln(x+1)^{\frac{1}{2}} - \ln(x)^{\frac{1}{2}} = \ln\left(\frac{(x+1)^{\frac{1}{2}}}{x^{\frac{1}{2}}}\right) = \ln\left(\sqrt{\frac{x+1}{x}}\right).$$

5. i c

$$\begin{aligned}
\ln\left(\frac{x^2\sqrt{x+1}}{\sqrt[3]{3x+4}}\right) &= \ln(x^2) + \ln(\sqrt{x+1}) - \ln(\sqrt[3]{3x+4})\\
&= 2\ln x + \ln(x+1)^{\frac{1}{2}} - \ln(3x+4)^{\frac{1}{3}}\\
&= 2\ln x + \frac{1}{2}\ln(x+1) - \frac{1}{3}\ln(3x+4).
\end{aligned}$$

ii c

$$\begin{aligned}\frac{d}{dx}\left(\ln\left(\frac{x^2\sqrt{x+1}}{\sqrt[3]{3x+4}}\right)\right) &= \frac{d}{dx}(2\ln x)+\frac{d}{dx}\left(\frac{1}{2}\ln(x+1)\right)-\frac{d}{dx}\left(\frac{1}{3}\ln(3x+4)\right)\\ &= \left(2\times\frac{1}{x}\right)+\left(\frac{1}{2}\times\frac{1}{x+1}\right)-\left(\frac{1}{3}\times\frac{1}{3x+4}\times 3\right)\\ &= \frac{2}{x}+\frac{1}{2(x+1)}-\frac{1}{3x+4}.\end{aligned}$$

6. iii

$$\frac{d}{dx}(\ln(x^2)) = 2\frac{d}{dx}(\ln x) = 2\times\frac{1}{x}=\frac{2}{x}.$$

v

$$\frac{d}{dx}(\cos x\times\ln x) = \cos x\times\frac{1}{x}+\ln x\times(-\sin x) = \frac{\cos x}{x}-\sin x\times\ln x.$$

vi

$$\frac{d}{dx}(\sqrt{\ln x}) = \frac{d}{dx}((\ln x)^{\frac{1}{2}}) = \frac{1}{2}(\ln x)^{-\frac{1}{2}}\times\frac{1}{x}=\frac{1}{2x\sqrt{\ln x}}.$$

7. Let P be the population of the city after t years. Then $P = Ae^{kt}$.
When $t = 0$ (in 1970) $P = 2\times 10^6$ so, $2\times 10^6 = Ae^0 = A$ ie $P = 2\times 10^6 e^{kt}$.

When $t = 10$ $P = 2.5\times 10^6$,

$$\begin{aligned}2.5\times 10^6 &= 2\times 10^6 e^{10k}\\ \frac{2.5}{2} &= e^{10k}\\ 10k &= \ln 1.25\\ k &= \frac{\ln 1.25}{10}\\ k &= 0.0223.\end{aligned}$$

Therefore, $P = 2\times 10^6 e^{0.022t}$.