Solutions to Selected Exercises Set 3

1. The answers are given in the notes.

2. **i** **a.** $(x+3)(x+3) = x(x+3) + 3(x+3) = x^2 + 3x + 3x + 9 = x^2 + 6x + 9.$

c. $(x+5)(x+5) = x^2 + 5x + 5x + 25 = x^2 + 10x + 25.$

f. $(x+a)(x+a) = x^2 + ax + ax + a^2 = x^2 + 2ax + a^2.$
This one gives us the general pattern.

$$(x+a)^2 = (x+a)(x+a) = x^2 + 2ax + a^2.$$

The $2ax$ is sometimes called the cross term.

ii **a.** $(x-3)(x-3) = x(x-3) - 3(x-3) = x^2 - 3x - 3x + 9 = x^2 - 6x + 9.$

d. $(x-4)^2 = (x-4)(x-4) = x^2 - 4x - 4x + 16 = x^2 - 8x + 16.$

f. $(x-a)^2 = (x-a)(x-a) = x^2 - ax - ax + a^2 = x^2 - 2ax + a^2.$
This one gives us the general pattern.

iii **a.** $(x+3)(x-3) = x(x-3) + 3(x-3) = x^2 - 3x + 3x - 9 = x^2 - 9.$

d. $(x-a)(x+a) = x(x+a) - a(x+a) = x^2 + ax - ax - a^2 = x^2 - a^2.$

This one gives us the pattern. It is often written as

$$x^2 - a^2 = (x+a)(x-a)$$

and is referred to as the difference of two squares.

iv **a.** $(x+3)(x+2) = x(x+2) + 3(x+2) = x^2 + 2x + 3x + 6 = x^2 + 5x + 6.$

b. $(x+3)(x-2) = x(x-2) + 3(x-2) = x^2 - 2x + 3x - 6 = x^2 + x - 6.$

c. $(x-3)(x+2) = x(x+2) - 3(x+2) = x^2 + 2x - 3x - 6 = x^2 - x - 6.$

d. $(x-3)(x-2) = x(x-2) - 3(x-2) = x^2 - 2x - 3x + 6 = x^2 - 5x + 6.$

The pattern exhibited by these examples can be summed up as follows:

a. and **d.**
If the signs in the brackets are the same then the constant is positive as the constant is the product of the constants in the brackets. Also, the coefficient of the cross term is sum of the constants in the brackets and has the same sign as them.

b. and **c.**
If the signs in the brackets are different then the constant is negative as the constant is the product of the constants in the brackets. Also, the coefficient of the cross term is the difference of the constants in the brackets and care must be taken to assign the correct sign to each one.

v These are very similar to the previous examples.

vi **a.** $(2x+1)(x+3) = 2x(x+3)+1(x+3) = 2x^2+6x+x+3 = 2x^2+7x+3.$

b. $(2x+1)(x-3) = 2x^2-6x+x-3 = 2x^2-5x-3.$

c. $(2x-1)(x+3) = 2x(x+3)-1(x+3) = 2x^2+6x-x-3 = 2x^2+5x-3.$

d. $(2x-1)(x-3) = 2x^2-6x-x+3 = 2x^2-7x+3.$

In the previous exercises we needed to take into account the coefficient of x^2 when we worked out the cross term.

3. We now need to use the patterns we spotted in the previous exercise to factorise the following exercises. We will some of each type.

a. x^2+4x+3

The $+3$ tells us that the signs in the brackets are the same, while the $+4x$ tells us that the sign must be $+$.

This gives us $(x+\cdot)(x+\cdot)$.

The product of the constants in the brackets equals 3, while their sum equals 4. Therefore 3 and 1 will do.

We get

$$x^2+4x+3 = (x+3)(x+1), \quad \text{which we check by expanding again.}$$

d. $x^2+7x+12$

$+12$ tells us that the signs are the same and $+7x$ tells us that they are $+$. The constants in the brackets must multiply to 12 and add to 7, so must be 6 and 1.

$$x^2+7x+12 = (x+6)(x+1).$$

f. $x^2-3x-10$

The -10 tell us that the signs in the brackets are different so we are looking for factors of -10 with a difference of -3. They must be -5 and 2.

$$x^2-3x-10 = (x-5)(x+2) \quad \text{again check by expanding.}$$

(Notice if we chose 5 and -2 we would get $x^2+3x-10$.)

j. x^2-x-12

-12 tells us that the signs in the brackets are different and we are looking for factors of -12 with a difference of -1. Take -4 and 3.

$$x^2-x-12 = (x-4)(x+3).$$

k. $x^2 - 11x + 10$

$+10$ tells us that the signs in the brackets are the same and $-11x$ tells us the signs are $-$. We are looking for factors of 10 whose sum is 11, so take 10 and 1.

$$x^2 - 11x + 10 = (x - 10)(x - 1).$$

m. $x^2 - 16x + 15$

$+15$ tells us that the signs in the brackets are the same and $-16x$ tells us that they are $-$. We are looking for factors of 15 whose sum is 16. Take 15 and 1.

$$x^2 - 16x + 15 = (x - 15)(x - 1).$$

s. $x^2 + 5x - 14$

We are looking for factors of -14 whose difference is 5, so take 7 and -2.

$$x^2 + 5x - 14 = (x + 7)(x - 2).$$

v. $m^2 + 6m + 9$

We are looking for factors of 9 with a sum of 6. Take 3 and 3.

$$m^2 + 6m + 9 = (m + 3)(m + 3) = (m + 3)^2.$$

w. $2x^2 + 7x + 3$

We know that the signs in the brackets are the same and must be $+$.

So we have $(2x + \cdot)(x + \cdot)$ as the only factors of 2 are 2 and 1.

We are looking for factors of 3 so try 3 and 1. Now try them in the bracket to see if we can get it to work.

$$(2x + 3)(x + 1) = 2x^2 + 5x + 3 \quad \text{which is not what we want.}$$

If we swop them over we get

$$(2x + 1)(x + 3) = x^2 + 7x + 3 \quad \text{which is what we were after.}$$

There is an element of trial and error in doing these exercises!

4. a.

$$\begin{aligned} x^2 - 7x + 12 &= 0 \\ (x - 6)(x - 1) &= 0 \end{aligned}$$

Therefore $x - 6 = 0$ or $x - 1 = 0$. That is, $x = 6$ or $x = 1$.

b.

$$\begin{aligned} x^2 + 3x &= 0 \\ x(x+3) &= 0 \end{aligned}$$

So, $x = 0$ or $x + 3 = 0$. That is, $x = 0$ or $x = -3$.

d.

$$\begin{aligned} 4x^2 - 4x &= 0 \\ 4x(x-1) &= 0 \end{aligned}$$

So, $x = 0$ or $x = 1$.

e.

$$\begin{aligned} x^2 - 4x - 5 &= 0 \\ (x-5)(x+1) &= 0 \end{aligned}$$

So, $x = 5$ or $x = -1$.

h.

$$\begin{aligned} 5x - x^2 &= 0 \\ x(5-x) &= 0 \end{aligned}$$

So, $x = 0$ or $5 - x = 0$, ie $x = 5$.

k.

$$\begin{aligned} x^2 - 81 &= 0 \\ (x+9)(x-9) &= 0 \end{aligned}$$

So, $x = -9$ or $x = +9$. This is the difference of two squares.

m.

$$\begin{aligned} x^2 - 1 &= 0 \\ (x+1)(x-1) &= 0 \end{aligned}$$

So, $x = -1$ or $x = +1$.

n.

$$\begin{aligned} 9x^2 - 16 &= 0 \\ (3x)^2 - 16 &= 0 \\ (3x+4)(3x-4) &= 0 \end{aligned}$$

So, $x = -\frac{4}{3}$ or $x = \frac{4}{3}$.

o.

$$\begin{aligned} x^2 - 2x + 1 &= 0 \\ (x-1)(x-1) &= 0 \end{aligned}$$

So, $x = 1$. Notice both brackets give the same answer.

r.

$$\begin{aligned} x^2 - 14x + 49 &= 0 \\ (x-7)(x-7) &= 0 \end{aligned}$$

So, $x = 7$.

s.

$$\begin{aligned} 10 + 3x - x^2 &= 0 \\ x^2 - 3x - 10 &= 0 \\ (x-5)(x+2) &= 0 \end{aligned}$$

So, $x = 5$ or $x = -2$. Notice we started by rearranging the equation to make the x^2 term positive.

u.

$$\begin{aligned} 2x^2 - 7x + 5 &= 0 \\ (2x-5)(x-1) &= 0 \end{aligned}$$

So, $x = \frac{5}{2}$ or $x = 1$.

v.

$$\begin{aligned} 6x^2 - 25x - 9 &= 0 \\ (2x-9)(3x+1) &= 0 \end{aligned}$$

So, $x = \frac{9}{2}$ or $x = -\frac{1}{3}$. It may take you several goes to get this one out as there is more than one way to factorise both 6 and 9.

5. For these next exercises we will use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

a. $x^2 - 3x - 5 = 0$

$$\begin{aligned} x &= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-5)}}{2(1)} \\ &= \frac{3 \pm \sqrt{9 + 20}}{2} \\ &= \frac{3 \pm \sqrt{29}}{2} \end{aligned}$$

That is $x = \frac{3+\sqrt{29}}{2}$ or $x = \frac{3-\sqrt{29}}{2}$.

c. $x^2 + 6x + 2 = 0$

$$\begin{aligned} x &= \frac{-6 \pm \sqrt{6^2 - 4(1)(2)}}{2(1)} \\ &= \frac{-6 \pm \sqrt{36 - 8}}{2} \\ &= \frac{-6 \pm \sqrt{28}}{2} \\ &= \frac{-6 \pm 2\sqrt{7}}{2} \qquad \sqrt{28} = \sqrt{4 \times 7} = 2\sqrt{7} \\ &= -3 \pm \sqrt{7} \end{aligned}$$

That is, $x = -3 + \sqrt{7}$ or $x = -3 - \sqrt{7}$.

e. $3x^2 - x - 3 = 0$

$$\begin{aligned} x &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-3)}}{2(3)} \\ &= \frac{1 \pm \sqrt{1 + 36}}{6} \\ &= \frac{1 \pm \sqrt{37}}{6} \end{aligned}$$

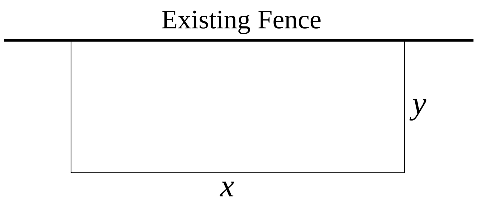
That is, $x = \frac{1+\sqrt{37}}{6}$ or $x = \frac{1-\sqrt{37}}{6}$.

f. $x^2 + 5x + 7 = 0$

$$\begin{aligned} x &= \frac{-5 \pm \sqrt{5^2 - 4(1)(7)}}{2(1)} \\ &= \frac{-5 \pm \sqrt{25 - 28}}{2} \\ &= \frac{-5 \pm \sqrt{-3}}{2} \end{aligned}$$

But here we have the $\sqrt{-3}$ which is not a real number, so there are no real solutions to this quadratic.

6. We approach these type of questions by drawing a picture and defining the variable we need.



Let x be the length of the side parallel to the existing fence and let y be the other side.

We have 300m of fencing material so $x + 2y = 300$. (1)

We want to fence an area of $10000m^2$, so $xy = 10000$. (2)

Now we need to eliminate either x or y to solve this equation.

Rearranging equation (1) we get $x = 300 - 2y$.

Sustituting in equation (2) we get $(300 - 2y)(y) = 10000$, ie $300y - 2y^2 = 10000$.

This gives us the quadratic $2y^2 - 300y + 10000 = 0$ which we can solve using the quadratic formula.

$$\begin{aligned} y &= \frac{300 \pm \sqrt{300^2 - 4(2)(10000)}}{2(2)} \\ &= \frac{300 \pm \sqrt{90000 - 80000}}{4} \\ &= \frac{300 \pm 100}{4} \end{aligned}$$

So, $y = \frac{300+100}{4} = 100$, or $y = \frac{300-100}{4} = 50$.

If $y = 100$, $x = 300 - 2(100) = 100$. If $y = 50$, $x = 300 - 2(50) = 200$.

So the dimensions of the rectangle are either 100m by 100m or 200m by 50m.

8. The rock will reach the ground when $s = 150$, so $150 = 5t + 4.9t^2$ or $4.9t^2 + 5t - 150 = 0$.

$$\begin{aligned} t &= \frac{-5 \pm \sqrt{5^2 - 4(4.9)(-150)}}{2(4.9)} \\ &= \frac{-5 \pm \sqrt{25 + 2940}}{9.8} \\ &= \frac{-5 \pm 54.45}{9.8} \end{aligned}$$

So, $t = 5.05$ or $t = -6.07$ (which is not possible given the physical situation).

Therefore the rock will take 5sec to reach the ground.