Solutions to Selected Exercises 4

1. i To verify that the given points lie on the parabola $y = x^2$ substitute in the value of x as follows:

For $A = (1, 1)$ when $x = 1$, $y = (1)^2 = 1$ as required so $A = (1, 1)$ lies on the parabola.

Similarly, for $C' = (0.9, 0.9801)$, when $x = 0.99$, $y = (0.99)^2 = 0.9801$ as required so $C' = (0.99, 0.9801)$ lies on the parabola.

ii The slopes of the chords are calculated using,

$$m = \frac{\text{change in } y}{\text{change in } x}.$$

So, for the chord AB is $m = \frac{1.21-1}{1.1-1} = \frac{0.21}{0.1} = 2.1$.

For the chord AB', $m = \frac{0.81-1}{0.9-1} = \frac{-0.19}{-0.1} = 1.9$.

For the chord AC, $m = \frac{1.0201-1}{1.01-1} = 2.01$.

For the chord AC', $m = \frac{0.9801-1}{0.99-1} = 1.99$.

iii It looks as if the slope of the parabola at A is going to be 2.

2. i The answers are given in the notes.

ii **b.** $f(x) = x^2 + x$

Since $a = 1 > 0$, the parabola is upright, and the function has a minimum value. The minimum values occurs when

$$f'(x) = 2x + 1 = 0.$$

That is, when $x = -\frac{1}{2}$ and hence $f(x) = (-\frac{1}{2})^2 - \frac{1}{2} = -\frac{1}{4}$ is the minimum value.

g. $f(x) = 4x - x^2$

Since $a = -1 < 0$, the parabola is upside down and the function has a maximum when $f'(x) = 0$.

$f'(x) = 4 - 2x = 0$ when $x = 2$.

Hence the maximum value of $f(x)$ is $f(x) = 4(2) - (2)^2 = 4$.

3. **c.** $f(x) = 4x^2 - 7x + 6$ so $f'(x) = 8x - 7$. Therefore $f'(0) = 8(0) - 7 = -7$.

4. **ii.** $y = x^2 - 5x + 6$

This parabola is upright and hence has a minimum when

$$\frac{dy}{dx} = 2x - 5 = 0.$$

That is, when $x = \frac{5}{2}$.

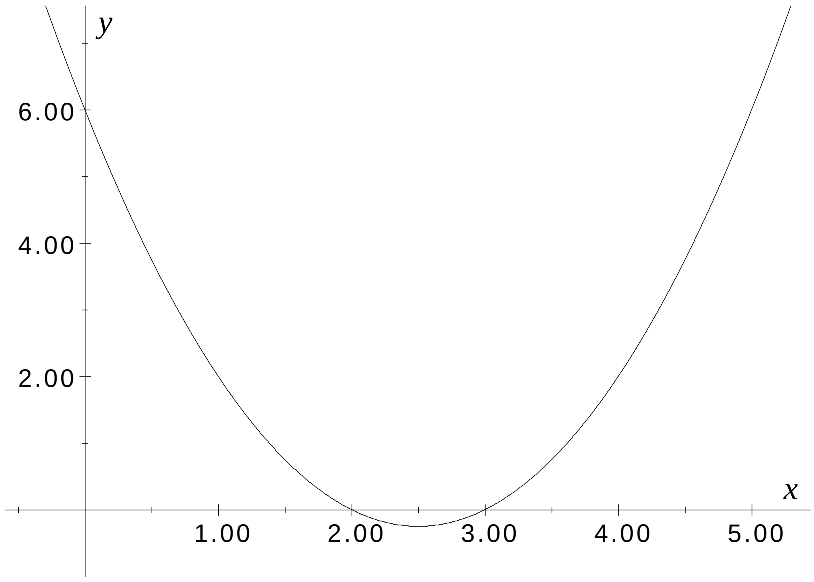
When $x = \frac{5}{2}$, $y = (\frac{5}{2})^2 - 5(\frac{5}{2}) + 6 = -\frac{1}{4}$.

It cuts the y axis when $x = 0$ ie at $y = 6$.

It cuts the x axis when $y = 0$ ie when $x^2 - 5x + 6 = 0$.

$$\begin{aligned} x^2 - 5x + 6 &= 0 \\ (x-3)(x-2) &= 0. \end{aligned}$$

That is, when $x = 2$ or $x = 3$.



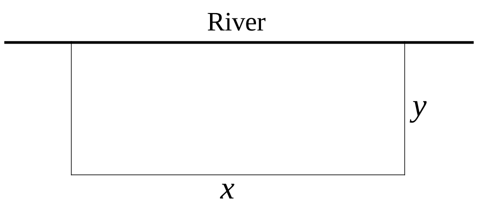
5. iii. To find the gradient of the curve at the given point we need to evaluate the derivative at that point.

$f'(x) = 2x - 3$. When $x = 0$, $f'(x) = -3$ so the gradient of the curve at the point $(0, 1)$ is -3.

The equation of the tangent to the curve at the point $(0, 1)$ is therefore $y = -3x + b$. When $x = 0$, $y = 1$ so $1 = -3(0) + b$ ie $b = 1$.

The equation of the tangent to the curve at $(0, 1)$ is $y = -3x + 1$ or $3x + y - 1 = 0$.

7.



Let x be the length of the side parallel to the river and let y be the length of the other side.

We have 300m of fencing material so $x + 2y = 300$.

The area of the field A is $A = xy$. Substituting $x = 300 - 2y$ we get,

$$A = (300 - 2y)y = 300y - 2y^2.$$

This is a quadratic with $a < 0$ so $A(y)$ has a maximum when $A'(y) = 0$.
Differentiating with respect to y, $A'(y) = 300 - 4y$.

When $A'(y) = 0$, $300 - 4y = 0$, so $y = 75$.

When $y = 75$, $x = 300 - 150 = 150$.

So the area of the field is maximised when the dimensions of the field are 150m by 75m.