Solutions to Selected Exercises 8

4. ii

$y = \dfrac{e^x + x}{2} = \dfrac{1}{2}(e^x + x)$ so $\dfrac{dy}{dx} = \dfrac{1}{2}(e^x + 1)$.

The derivative of e^x is e^x and the derivative of x is 1.

Also, note that we have used the rule that the derivative of a constant $\times$ a function is equal to the constant $\times$ the derivative of the function. (See page 15 of the notes.)

5. $P = Ae^{0.02t}$

When $t = 0$ (in 1970) $P = 100,000$ so substituting we get $100000 = Ae^{(0.02)(0)} = Ae^0 = A$.

So, $P = 100000e^{0.02t}$.

In 1980 $t = 10$, so $P = 100000e^{(0.02)(10)} = 100000(1.2214) = 122140$.

Therefore, the population in 1980 will be 122,000 to the nearest thousand.

6. ii $A = A_0e^{-0.00012t}$

When $t = 6000$, $A = A_0e^{(-0.00012)(6000)} = A_0e^{-0.72} = 0.49A_0$.

There is about 0.5 of the initial amount remaining after 6000 years.

7. iii $(x^5 + 1)^3$

Let $u = f(x) = x^5 + 1$ and $g(u) = u^3$, then $g(f(x)) = g(x^5 + 1) = (x^5 + 1)^3$.

iv $\dfrac{1}{3 - e^x}$

Let $u = f(x) = 3 - e^x$ and $g(u) = \dfrac{1}{u}$, then $g(f(x)) = g(3 - e^x) = \dfrac{1}{3 - e^x}$.