Probability, Arrow of Time and Decoherence

Guido Bacciagaluppi
IHPST, Paris

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Plan of talk:

1. Introduction
2. Decoherence and decoherent histories
3. Time (a)symmetry and decoherent histories
4. Time (a)symmetry and probabilities

1. Introduction

Two questions:
(A) time-asymmetry of the concept of probability,
(B) arrow of time in QM.

(A) Probabilities used in time-asymmetric ways:

deliberation,
evolution of chances by conditionalisation upon past events,
open future, fixed past...

BUT:

Time asymmetry not necessarily present in (or justified by?) the formalism.

E.g. a classical stochastic process is defined time-symmetrically as a probability measure over a space of trajectories.

Here: case of quantum probabilities,
specifically in the context of decoherence
(decoherent histories).

(B) Arrow of time in QM:

The Schrödinger equation is time-symmetric if one implements time reversal as

$$\psi(x,t) \mapsto \psi^*(x,-t). \tag{1}$$

Obviously, not the same as saying all solutions are time-symmetric.

A solution is time-symmetric iff

$$\psi(x,t) = \psi^*(x,-t) \tag{2}$$

for all times t.

Collapse is not time-symmetric, in particular collapse probabilities are not:

$$\mathrm{Tr}\Big(P_{\alpha_n}U_{t_nt_{n-1}}\dots P_{\alpha_1}U_{t_1t_0}|\psi(t_0)\rangle\langle\psi(t_0)| \\ U^*_{t_1t_0}P_{\alpha_1}\dots U^*_{t_nt_{n-1}}P_{\alpha_n}\Big) \tag{3}$$

is different from

$$\mathrm{Tr}\Big(P_{\alpha_1}U_{t_1t_2}\dots P_{\alpha_n}U_{t_nt_{n+1}}|\psi(t_{n+1})\rangle\langle\psi(t_{n+1})| \\ U^*_{t_nt_{n+1}}P_{\alpha_n}\dots U^*_{t_1t_2}P_{\alpha_1}\Big), \tag{4}$$

nor are the corrsponding sequences of states reversed.

If collapse is a real physical process, it appears we have time asymmetry of fundamental laws.

If one does NOT like that, two options:
(a) symmetrise collapse,
(b) take a no-collapse approach.

(a) Symmetrisation of collapse:

Can use the ABL formula (Aharonov, Bergmann and Lebowitz):

$$\frac{\mathrm{Tr}\Big(|\psi(t_{n+1})\rangle\langle\psi(t_{n+1})|U_{t_{n+1}t_n}P_{\alpha_n}U_{t_nt_{n-1}}\dots P_{\alpha_1}U_{t_1t_0} |\psi(t_0)\rangle\langle\psi(t_0)|U^*_{t_1t_0}P_{\alpha_1}\dots U^*_{t_nt_{n-1}}P_{\alpha_n}U^*_{t_{n+1}t_n}\Big)}{\sum_{\alpha_1,\dots,\alpha_n}\mathrm{Tr}\Big(|\psi(t_{n+1})\rangle\langle\psi(t_{n+1})|U_{t_{n+1}t_n}P_{\alpha_n}U_{t_nt_{n-1}}\dots P_{\alpha_1}U_{t_1t_0} |\psi(t_0)\rangle\langle\psi(t_0)|U^*_{t_1t_0}P_{\alpha_1}\dots U^*_{t_nt_{n-1}}P_{\alpha_n}U^*_{t_{n+1}t_n}\Big)}, \tag{5}$$

interpreting it not in terms of pre- and post-selection, but in terms of a two-vector formalism (Aharonov and Vaidman).

(b) No-collapse approaches (Bohm, Everett):

Time-symmetric Schrödinger equation is fundamental.

Only appearence of collapse and of time asymmetry.

Such approaches arguably use *decoherence*.

2. Decoherence and decoherent histories

Taking decoherence as part of the formalism of QM (i.e. only uncontroversial aspects).

Decoherence: the study of when interference is suppressed (through suitable interaction with some environment).

E.g. two-slit experiment with suitable interaction taking place between the slits and the screen: interference pattern is suppressed.

In particular, probabilities for detection at the screen can be calculated from probabilities for detection at the slits and probabilities of detection at the screen conditional on detection at the slits.

Remarks:

It does not follow that the particle went through one of the slits (nor from similar applications of decoherence that measurements have outcomes).
But: one can formally assign probabilities to sequences of events (histories).
The effect depends on special (disentangled) initial conditions.

Uncontroversial: these probabilities play a role at the phenomenological level.

I would claim: no-collapse approaches (Bohm, Everett...) use decoherence, and these probabilities simply *are* the probabilities emerging in these approaches.

But no need to argue that for present purposes.

Today only discuss formal features of these probabilities.

Decoherent histories (e.g. Gell-Mann and Hartle, Griffiths, Omnès).

Again bracketing controverial aspects: I take it that the *formalism* of decoherent histories can be used as an interpretationally neutral description of certain features of decoherence that are of interest here.

Namely: abstract definition of decoherence in terms of when we can consistently assign probabilities to alternative histories (sequences of projection operators).

Details as follows.

Take orthogonal families of projections with

$$\sum_{\alpha_1} P_{\alpha_1}(t) = \mathbf{1}, \ldots, \sum_{\alpha_n} P_{\alpha_n}(t) = \mathbf{1} \tag{6}$$

(in Heisenberg picture).

Choose times $t_1, \ldots, t_n$.

Define *histories* as time-ordered sequences of projections

$$P_{\alpha_1}(t_1), \ldots, P_{\alpha_n}(t_n). \tag{7}$$

Given a state ρ, we wish to define probabilities for the set of histories.

The usual probability formula (based on the projection postulate) is

$$\mathrm{Tr}\Big(P_{\alpha_n}(t_n)\ldots P_{\alpha_1}(t_1)\rho P_{\alpha_1}(t_1)\ldots P_{\alpha_n}(t_n)\Big). \qquad (8)$$

But in general it is not valid for subhistories (i.e. subsequences for some $t_{i_1},\ldots,t_{i_m}, m<n$): the corresponding *marginals* do not have the same form, because of interference terms.

Consistency or (weak) *decoherence* condition: precisely that interference terms should vanish for any pair of distinct histories,

$$\mathrm{Re}\mathrm{Tr}\Big(P_{\alpha'_n}(t_n)\ldots P_{\alpha'_1}(t_1)\rho P_{\alpha_1}(t_1)\ldots P_{\alpha_n}(t_n)\Big)=0. \qquad (9)$$

We seem to have embedded the time-asymmetric probabilities (8) in a time-symetric theory with no collapse.

3. Time (a)symmetry and decoherent histories

However, Hartle for one is not satisfied with this analysis of time (a)symmetry. Why not?

The above probabilities are time-asymmetric in the sense that the state ρ enters the probability formula as an 'initial' state,

$$\mathrm{Tr}\Big(P_{\alpha_n}(t_n)\ldots P_{\alpha_1}(t_1)\rho P_{\alpha_1}(t_1)\ldots P_{\alpha_n}(t_n)\Big). \tag{10}$$

But they are defined thanks to a similarly asymmetric decoherence condition, namely

$$\mathrm{Re}\mathrm{Tr}\Big(P_{\alpha'_n}(t_n)\ldots P_{\alpha'_1}(t_1)\rho P_{\alpha_1}(t_1)\ldots P_{\alpha_n}(t_n)\Big) = 0. \tag{11}$$

According to Hartle, this means putting in the time asymmetry by hand.

Hartle suggests introducing a new decoherence condition, in which there appear *two* states ρ_i and ρ_f, one in 'initial' position and one in 'final' position:

$$\mathrm{Re}\mathrm{Tr}\Big(\rho_f P_{\alpha_n'}(t_n)\dots P_{\alpha_1'}(t_1)\rho_i P_{\alpha_1}(t_1)\dots P_{\alpha_n}(t_n)\Big) = 0. \tag{12}$$

Correspondingly, the probability for a history becomes

$$\frac{\mathrm{Tr}\Big(\rho_f P_{\alpha_n}(t_n)\dots P_{\alpha_1}(t_1)\rho_i P_{\alpha_1}(t_1)\dots P_{\alpha_n}(t_n)\Big)}{\mathrm{Tr}(\rho_f\rho_i)}. \tag{13}$$

(That is, provided $\mathrm{Tr}(\rho_f\rho_i)\neq 0$.)

These formulas now are explicitly time-symmetric, similarly to the ABL formula (with a simplified normalisation factor).

The observed time-asymmetric phenomena, according to Hartle, are to be explained in terms of a contingent asymmetry between the 'initial' and 'final' boundary conditions, with ρ_i a certain kind of pure state and ρ_f close to the identity operator.

Criticism:

Note that this is a generalisation of QM: it leads to different predictions if ρ_f is not close to the identity. So far it is not independently motivated.

Hartle remarks it may be useful to describe violations of unitarity. But why not then take Aharonov and Vaidman's two-vector formalism?

The introduction of ρ_f seems to defeat the purpose of the no-collapse strategy. Was not the Schrödinger equation already supposed to give us time symmetry?

Alternative proposal:

No need to go to a more general theory and introduce an asymmetry between ρ_i and ρ_f.

The usual theory is already symmetric enough. The asymmetry (if any) is encoded in the special solution ρ, as follows.

For a given state and set of histories we may have:

$$\mathrm{ReTr}\Big(P_{\alpha_n'}(t_n)\ldots P_{\alpha_1'}(t_1)\rho P_{\alpha_1}(t_1)\ldots P_{\alpha_n}(t_n)\Big) = 0 \tag{14}$$

but at the same time we may have:

$$\mathrm{ReTr}\Big(P_{\alpha_1'}(t_1)\ldots P_{\alpha_n'}(t_n)\rho P_{\alpha_n}(t_n)\ldots P_{\alpha_1}(t_1)\Big) \neq 0. \tag{15}$$

That is, just as we can ask whether the 'forward' decoherence condition (14) is satisfied (this is an objective feature of the state ρ and the given set of histories), we can equally well ask whether for the same ρ and the set of the histories with *opposite* time ordering the interference terms vanish ('backwards' decoherence condition).

Either condition allows us to define probabilities for sets of histories.

Since individual solutions to the Schrödinger equation need not be symmetric, it is perfectly possible that for a given a set of histories, a state ρ will make this set decoherent in one direction of time but not the other.

Should *both* conditions be satisfied, then one can show that

$$\begin{aligned}\mathrm{Tr}\Big(P_{\alpha_n}(t_n)\dots P_{\alpha_1}(t_1)\rho P_{\alpha_1}(t_1)\dots P_{\alpha_n}(t_n)\Big) &= \\ = \mathrm{Re}\mathrm{Tr}\Big(P_{\alpha_n}(t_n)\dots P_{\alpha_1}(t_1)\rho\Big)\end{aligned} \tag{16}$$

as well as

$$\begin{aligned}\mathrm{Tr}\Big(P_{\alpha_1}(t_1)\dots P_{\alpha_n}(t_n)\rho P_{\alpha_n}(t_n)\dots P_{\alpha_1}(t_1)\Big) &= \\ = \mathrm{Re}\mathrm{Tr}\Big(\rho P_{\alpha_n}(t_n)\dots P_{\alpha_1}(t_1)\Big).\end{aligned} \tag{17}$$

But the two right-hand sides are equal, by cyclicity of the trace.

Thus, the two sets of probabilities that can be defined for the given set of histories coincide.

4. Time (a)symmetry and probabilities

Use the above proposal to analyse the status of probabilities in the context of (decoherence-based) no-collapse approaches to QM.

Will obtain general claims valid even when one fills in details about the interpretation of probabilities (which is different in different interpretations of QM).

Three such claims:

(a) The fundamental level (Schrödinger equation and the pair of decoherence conditions) is perfectly time-symmetric.

(b) From this fundamental level emerge genuinely time-asymmetric probabilistic laws at the level of the histories.

(c) This emergence of time asymmetry is nevertheless merely contingent.

(a) Our analysis restores perfect time symmetry at the level of the Schrödinger equation and the possible conditions that allow one to define probabilities.

Indeed, a set of histories could satisfy either the forward or the backward decoherence condition (or neither or both) with respect to a given state.

In either case one gets formally time-asymmetric probabilities, like the usual collapse probabilities, but they could be equally well 'forward'-directed as 'backward'-directed.

(b) Do such formally asymmetric probabilities give rise to genuinely time-asymmetric behaviour at the level of the histories?

Worry: 'Boltzmann's time-bomb' (Price). If the time asymmetry depends on special initial conditions (the wave function is initially disentangled), could it not be trumped by special final conditions (the wave function is disentangled at both temporal ends)?

Indeed, what happens if we take a time-symmetric solution

$$\psi(x,t) = \psi^*(x,-t)\ ? \tag{18}$$

If $\psi(x,t)$ is symmetric as in (2), then for each set of histories

$$P_{\alpha_1}(t_1), \ldots, P_{\alpha_n}(t_n) \tag{19}$$

satisfying the forward decoherence condition there will be a corresponding set

$$P_{\alpha_1}(-t_1), \ldots, P_{\alpha_n}(-t_n) \tag{20}$$

satifying the backward decoherence condition.

However, in general these sets *cannot* be combined into a single set satisfying either condition.

E.g., taking all $t_i < 0$, we have decoherence from either temporal end towards 0, or (from either perspective) decoherence followed by recoherence: no 'consistent extension' beyond 0.

This does not mean observable 'collapse' behaviour followed by observable 'anti-collapse' behaviour: it means quantum erasure of all records of the previous collapses.

Thus the temporally asymmetric behaviour cannot be trumped by appropriate boundary conditions at the opposite temporal end. In this sense, it is genuinely asymmetric.

This justifies giving a qualitatively different status to 'forward' transition probabilities (a kind of objectivity relating to an open future) and to 'backward' transition probabilities (a merely epistemic status relating to a fixed past), however these be interpreted in the first place.

(One can give intuitively appealing arguments if one brings in a specific interpretation of QM: the asymmetry of the probabilities corresponds to asymmetry of branching structure in Everett, or to asymmetry of empty-wave formation in Bohm.)

(c) Nevertheless, this time asymmetric behaviour is only contingent.

Indeed, there is another sense in which a contingent symmetry of the solution can prevent the emerging probabilities from being time-asymmetric: if both decoherence conditions are satisfied.

In this case, the probabilities can be defined using either the 'forward' or the 'backward' expression, and the formal time asymmetry disappears.

Presumably, this is the case when the quantum description reduces to a classical stochastic process (and the discussion of probabilities in the classical context applies even to the quantum probabilities).