The Role of the Observer in Quantum Computation

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Outline

1. The ‘black-box’ view of measurement
2. What is it to be ‘an observer of X’?
3. Application to the EPR scenario
4. One-way quantum computation
5. Observation in one-way quantum computation

1 The 'black-box' view of measurement

Bub's argument

> "[A] mechanical theory that purports to solve the measurement problem is not acceptable if it can be shown that, in principle, the theory can have no excess empirical content over a quantum theory. By the CBH theorem, given the information-theoretic constraints any extension of a quantum theory ... must be empirically equivalent to a quantum theory, so no such theory can be acceptable as a deeper mechanical explanation of why quantum phenomena are subject to the information-theoretic constraints. To be acceptable, a mechanical theory that includes an account of our measuring instruments as well as the quantum phenomena they reveal (and so purports to solve the measurement problem) must violate one or more of the information-theoretic constraints."

Bub (2004)

CBH $\Rightarrow$

INFO $\Rightarrow$ any mechanical account of measurement is empirically equivalent to 'standard' quantum theory

The Information-Theoretic Principles

(Just to satisfy your curiosity. . .)

INFO says (roughly):

- No superluminal transfer of information
- No unconditionally secure bit commitment
- No cloning

Bub's Conclusion

Bub concludes:

> "[T]he rational epistemological stance is to suspend judgement about all these empirically equivalent but necessarily underdetermined theories and regard them all as unacceptable. It follows that our measuring instruments ultimately remain black boxes at some level that we represent in the theory simply as probabilistic sources of ranges of labelled events or 'outcomes'..."

Bub (2004)

Don't try to provide an account of measurements (observation).

Two Objections

Why believe INFO?

It seems to me that the only really powerful reasons that we have for believing in INFO come from quantum theory itself.

No account of measurement?

From the fact that we 'cannot' incorporate measurement entirely within the theory, it does not follow that we can say *nothing* about measurement, that measurements are black-boxes.

This talk is, in part, supposed to show that we can say a lot about measurement (observation), and that doing so is in fact important *even* if our primary interest is in standard quantum theory understood as a theory of information (and computation).

2 Being 'an observer of X'

Defining Observables Group-Theoretically

(Many) quantum-theoretic observables can be *defined* in terms of the invariances and covariances that they obey.

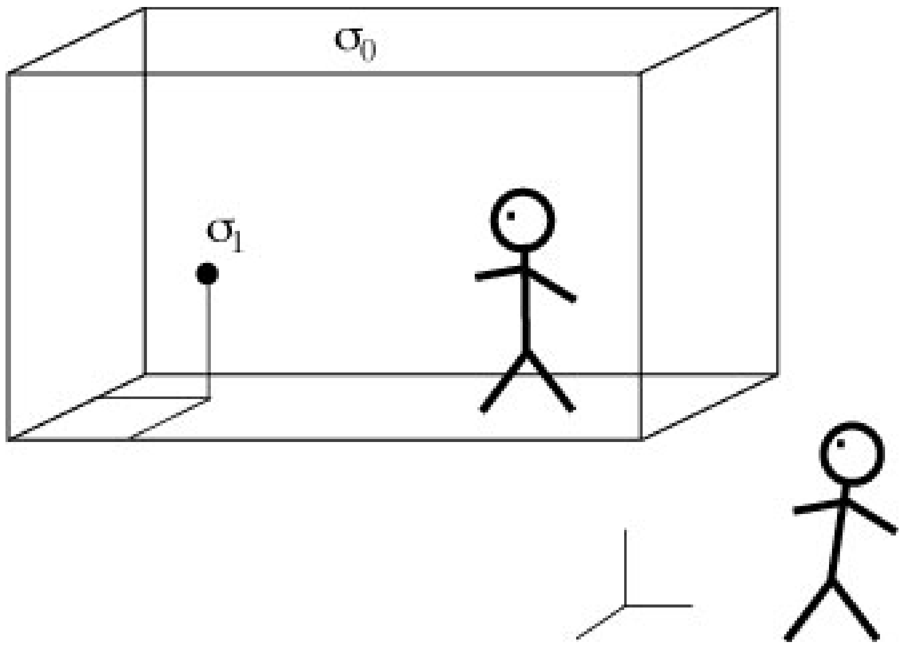
Example:

$$U_{\vec{a}} E^Q(\Delta) U_{\vec{a}}^{-1} = E^Q(\alpha_{\vec{a}} \Delta)$$

$$V_{\vec{b}} E^Q(\Delta) V_{\vec{b}}^{-1} = E^Q(\Delta)$$

- Mackey's Imprimitivity Theorem is in the background, here.
- We can make covariance into invariance by redefining the observable relationally
- We are ignoring, here, other arguably essential properties of observers, such as the capacity to 'record' the result of the observation.

'Internal' and 'External' Observers



Coordinates The internal observer measures position by

means of operators Q_n^{int} $(n > 0)$ for the systems σ_n. The external observer measures position by means of the operators Q_n^{ext} $(n \geq 0)$.

The external observer's task is to describe, in his own ('absolute') language what the internal observer is 'really' measuring.

The Role of Inertial Frames

- We require that the invariances hold *in some purportedly 'inertial' frame*, a frame in which the law of motion is true.
- This requirement leads to a rewriting of the invariance requirements in terms of conservation principles.

Requirement for being an observer of X

F (a frame of reference, as determined, in some specified way, by a physical system) is an *observer of* X (during the time Δt) if one can define, in F, some observable, $\hat{X}$, such that the transformation of $\hat{X}$ to some inertial frame (during Δt) is a relational observable satisfying the invariances that are definitive of X.

Example: Position and Momentum

Transformation from a frame in which observables are (necesarily!) defined relationally, to some 'absolute' frame:

$$U_{\mathsf{AK}} = \exp\left(-i\sum_{n>0} P_n^{\mathsf{ext}} Q_0^{\mathsf{ext}}\right)$$

Transforming Q_1^{int} to the external frame via U_{AK} yields the expected result:

$$U_{\mathsf{AK}} Q_1^{\mathsf{int}} U_{\mathsf{AK}}^{-1} = Q_1^{\mathsf{ext}} - Q_0^{\mathsf{ext}}.$$

Now consider what happens during a measurement of position by means of a 'pointer' observable, Q_2 on system 2, conjugate to P_2. The 'interior' observer writes:

$$H_{\mathsf{I}}^{\mathsf{int}} = g(t) Q_1^{\mathsf{int}} \otimes P_2^{\mathsf{int}}$$

Transforming to the 'external' frame:

$$H_{\mathsf{I}}^{\mathsf{ext}} = g(t)(Q_1^{\mathsf{ext}} - Q_0^{\mathsf{ext}}) \otimes P_2^{\mathsf{ext}}$$

which is what one would expect.

Disturbance of Momentum

Note that $[H_{\mathsf{I}}^{\mathsf{ext}}, P_0^{\mathsf{ext}}] \neq 0$.

- A measurement of the position of particle 1 'disturbs' the momentum of the *frame previously used to define its momentum*
- A observer 'in frame F' *cannot* account for this change.
- Hence, for such an observer, *momentum is not conserved* (during the interaction)
- Hence, such an observer is not an observer of the momentum of particle 1.

Of course, we can always move to an enveloping frame, in which the change in the momentum of system 0 can be detected.

3 Application to the EPR Scenario

One often assumes that it is a trivial matter to measure Q_1 and P_2 on an EPR pair. After all, they commute!

But matters are not so simple, if we are presuming that P_2 is defined relative to the same frame regardless of the measurement of Q_1 then more must be said: *we must move to the enveloping frame*, in which the effect of the measurement of Q_1 on system 0 can itself be measured.

Spin, Angle, and Direction

See, e.g., Busch, Grabowski, and Lahti (1995)

One may define an angle observable by piggy-backing on the position observable, as follows.

Angle: Piggy-backing on Position Let S be the half-open interval of angles, $[0, 2\pi)$. Let

$$E^A : [0, 2\pi) \to \mathcal{L}(H)$$

be a POV measure. It must be 'covariant' under rotations.

For $\Theta \in \mathcal{B}(0, 2\pi)$, define

$$\Delta_\Theta = \left\{ \vec{x} \in \mathbb{R}^3 \,\middle|\, \frac{x_1}{\sqrt{x_1^2 + x_2^2}} = \cos\varphi, \frac{x_2}{\sqrt{x_1^2 + x_2^2}} = \sin\varphi, \varphi \in \Theta \right\}$$

Then we can define a PV measure:

$$E^A : \Theta \in \mathcal{B}(0, 2\pi) \to E^X(\Delta_\Theta)$$

Side Remark: The E^A are multiplication operators on the space

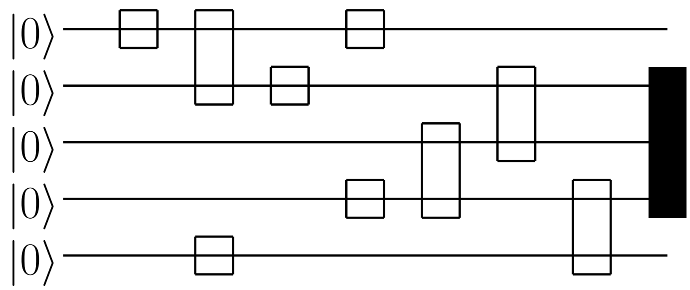
$$L^2\Big(\mathbb{R}^+ \times [0, \pi] \times [0, 2\pi)\Big)$$

and they are conjugate to L_z, (obey the Weyl relations).

4 One-way Quantum Computation

'Traditional' Quantum Computation

The 'traditional' picture of quantum computation:



- *many* particles
- gates are all unitary
- only one measurement is involved (at the end)

One-Way Quantum Computation—notation

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \text{ (the 'computational basis')}$$

$$|\pm\rangle = |0\rangle \pm |1\rangle \quad \text{(ignoring normalization)}$$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \text{(unitary)}$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \text{(unitary)}$$

$$CZ|i\rangle|j\rangle := CZ|ij\rangle = (-1)^{ij}|i\rangle|j\rangle \qquad \text{'controlled phase'}$$

A Simple One-Way Quantum Computation

Initial Preparation:

$$
\begin{aligned}
|\psi\rangle|+\rangle|+\rangle \;&\rightarrow CZ_{12}|\psi\rangle|+\rangle|+\rangle \\
&= CZ_{12}(a|0\rangle + b|1\rangle)(|0\rangle + |1\rangle)|+\rangle \\
&= CZ_{12}(a|00\rangle + a|01\rangle + b|10\rangle + b|11\rangle)|+\rangle \\
&= (a|00\rangle + a|01\rangle + b|10\rangle - b|11\rangle)|+\rangle
\end{aligned}
$$

'Projective' Measurement in the basis
$M(\theta) = \{|0\rangle \pm e^{i\theta}|1\rangle\}$ (with the result $|0\rangle + e^{i\theta}|1\rangle$):

$$
\begin{aligned}
&\rightarrow \;(|0\rangle + e^{i\theta}|1\rangle)(a|+\rangle + e^{-i\theta}|-\rangle)|+\rangle \\
&= \;(|0\rangle + e^{i\theta}|1\rangle)\, W(-\theta)|\psi\rangle|+\rangle
\end{aligned}
$$

with (again, up to a normalizing factor)

$$
W(\theta) = \begin{pmatrix} 1 & e^{i\theta} \\ 1 & -e^{i\theta} \end{pmatrix}
$$

(Cf.: computation by 'gate teleportation'.)

Continuing the One-Way Quantum Computation

Now we can entangle particle 2 with particle 3:

$$\begin{aligned} \rightarrow\ & |\text{blah}\rangle\ \ CZ_{23}(a|+\rangle + e^{-i\theta}b|-\rangle)|+\rangle \\ =\ & |\text{blah}\rangle\ \ \Big[a(|00\rangle + |01\rangle + |10\rangle - |11\rangle) \\ & \qquad\quad + e^{-i\theta}b(|10\rangle + |01\rangle - |10\rangle + |11\rangle)\Big] \end{aligned}$$

and measure in the basis $M(\phi)$ (presuming, again, the result $\{|0\rangle + e^{i\phi}|1\rangle$):

$$\rightarrow |\text{blah}\rangle|\text{blah}\rangle\, W(-\theta)W(-\phi)|\psi\rangle$$

NB: The entangling operations commute with the measurements (on the input state), so we can do all of the entangling at once, thus creating a highly entangle 'cluster state'.

5 The 'Issue' of Observation in 1WQC

Simulating Arbitrary Gates via Measurement

Universality
The set of gates $\{CZ\} \cup \{W(\theta) | \text{for all } \theta\}$ is universal for quantum computation.

> Any quantum computation can be performed via an appropriate preparation of a cluster state, plus projective measurements on single particles in the bases $M(\theta) = \{|0\rangle \pm e^{i\theta}|1\rangle\}$. *These operations commute on the input state.*

Keeping track of results...

If the result of our earlier original measurement on were $|0\rangle - e^{i\theta}|1\rangle Z$, then the state is:

$$(|0\rangle - e^{i\theta}|1\rangle) ZW(-\theta)|\psi\rangle|+\rangle$$

In general, after completely preparing the cluster state (by applying CZ_{12} and CZ_{23}, and measuring in the basis $M(\theta) = \{|0\rangle \pm e^{i\theta}|1\rangle\}$ on particle 1, then $M(\theta) = \{|0\rangle \pm e^{i\theta}|1\rangle\}$ for particle 2, we end up with:

$$|\text{blah}\rangle|\text{blah}\rangle Z^{s_1} W(-\theta) Z^{s_2} W(-\phi)|\psi\rangle$$

with $s_n \in \{0, 1\}$.

> In general we must keep track of the results of measurements, and 'adapt' later measurements to the earlier results.

... and consequences

But matters are worse, because the $M(\theta)$ *must* be defined *relationally*. But, by considerations similar to those entertained earlier, measuring in the basis $M(\theta)$ will disturb the frame that defines a direction in space. How, then, do we recover what is meant by $M(\phi)$, the basis for the *next* measurement?

The measurements in 1WQC are 'adaptive' in *two* senses:

- Results of earlier measurements can require one to change the basis defining later measurements
- The effect of a measurement in the basis $M(\theta)$ on the reference frame itself must be 'tracked' in an enveloping reference frame, in order to know what $M(\phi)$ means at all.

Lesson

1WQC is hard!

References